FOM 12 **Lesson 5.1 – Exploring Probability** Mrs. Dawe

Experimental Probability

The probability of event A, where n(A) is the number of times event A occurred and n(T) is the total number of trials, T, in the experiment.

event occurs

$$P(A) = \frac{n(A)}{n(T)}$$

trials

Theoretical Probability

The probability of event A where n(A) is the number of favourable outcomes for the event A and n(S) is the total number of outcomes in the sample space, S, where all outcomes are equally likely.

event happening

$$P(A) = \frac{n(A)}{n(S)}$$

all events

Things to know:

- A game is fair when all players are equally likely to win
- An event is a collection of outcomes that satisfy a specific condition
- The probability of an event can range from 0 (impossible) to 1 (certain)
- You can express probability as a fraction, a decimal, or a percent

FOM 12 **Lesson 5.2 – Probability and Odds** Mrs. Dawe

Odds express a level of confidence about the occurrence of an event.

Odds in favour is the ratio of the probability that an event will occur to the probability that the event will not occur (favourable outcomes to unfavourable outcomes):

$$\frac{P(A)}{P(A')}$$

$P(A) : P(A')$

Odds against is the ratio of the probability that an event will not occur to the probability that the event will occur (unfavourable outcomes to favourable outcomes):

$$\frac{P(A')}{P(A)}$$

$P(A') : P(A)$

$P(A')$ is the probability of the complement of A:

$$P(A') = 1 - P(A)$$

* opposite

Examples:

1. Bailey holds all the hearts from a standard deck of 52 cards. He asks Morgan to choose a single card without looking. Determine the odds in favour of Morgan choosing a face card.

$C = \{J, Q, K\}$ $C' = \{A, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

3:10

2. A computer randomly selects a university student's name from the university database to award a $100 gift certificate for the bookstore. The odds against the selected student being male are 57:43. Determine the probability that the randomly selected university student will be male.

$57 + 43 = 100$ total

female male

$P(\text{male}) = \frac{43}{100} = 43\%$

3. A group of grade 12 students are holding a charity carnival to support a local animal shelter. The students have created a dice game that they call Bim and a card game that they call Zap. The odds against winning Bim are 5:2, and the odds against winning Zap are 7:3. Which game should Madison play?

$5 + 2 = 7$ outcomes

$7 + 3 = 10$ outcomes

$P(\text{winning Bim}) = \frac{2}{7} = 0.285$

$P(\text{winning Zap}) = \frac{3}{10} = 0.3$

Use the **Fundamental Counting Principle** and techniques involving **permutations** and **combinations** to solve probability problems with many possible outcomes. The context of each particular problem will determine which counting technique you will use.

— — — / $_nP_r$ / $_nC_r$ / !

Examples:

1. Jason, Ethan, and Albert are competing with seven other boys to be on their school's cross-country team. All the boys have an equal chance of winning the trial race. Determine the probability that Jason, Ethan and Albert will place first, second, and third, in any order.

$3 \cdot 2 \cdot 1$ = 6 outcomes

$10 \cdot 9 \cdot 8$ = 720 outcomes possible

$\Rightarrow \frac{6}{720} = \frac{1}{120} = 0.83\%$

2. About 20 years after they graduated from high school, Blake, Mario, and Simon met in a mall. Blake had two daughters with him, and he said he had three other children at home. Determine the probability that at least one of Blake's children is a boy.

$2 \cdot 2 \cdot 2$ = 8 outcomes

G G G is only outcome with no boys

$\frac{1}{8}$ no boys $\rightarrow \frac{7}{8}$ for at least one boy / 87.5%

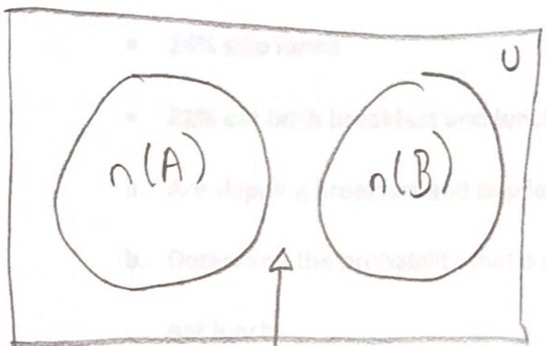
3. Beau hosts a morning radio show in Saskatoon. To advertise his show, he is holding a contest at a local mall. He spells out SASKATCHEWAN with letter tiles. Then he turns the tiles face down and mixes them up. He asks Sally to arrange the tiles in a row and then turn them face up. If the row of tiles spells SASKATCHEWAN, Sally will win a new car. Determine the probability that Sally will win the car.

$$\frac{12!}{2! \cdot 3!} = 39\ 916\ 800 \text{ outcomes}$$

*only one way to properly spell SASKATCHEWAN

$$P(\text{winning}) = \frac{1}{39\ 916\ 800}$$

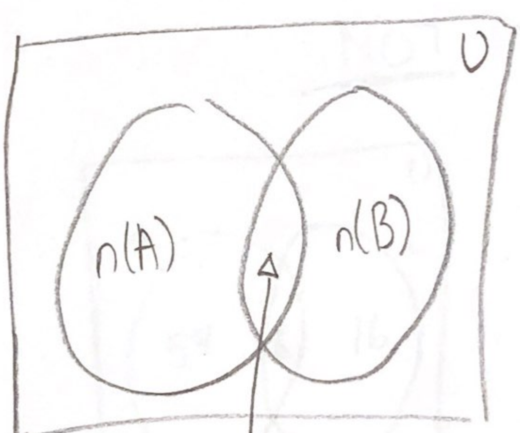
You can represent the favourable outcomes of two mutually exclusive events, A and B, ~~are~~ as two disjoint sets. For example:



$n(A \cap B) = 0$
(no common elements)

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

You can represent the favourable outcomes of two non-mutually exclusive events, A and B, as two intersecting sets. For example:



$n(A \cap B)$ (common elements)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

or

$$P(A \cup B) = P(A \setminus B) + P(B \setminus A) + P(A \cap B)$$

Examples:

1. A school newspaper published the results of a recent survey.

   Eating Habits: Student Survey Results

   - **62% skip breakfast**
   - **24% skip lunch**
   - **22% eat both breakfast and lunch**

   a. Are skipping breakfast and skip lunch mutually exclusive events?

   b. Determine the probability that a randomly selected student skips breakfast but not lunch.

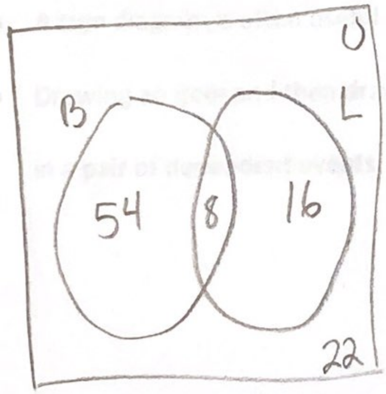
   c. Determine the probability that a randomly selected student skips at least one of the breakfast or lunch.

a) To be mutually exclusive the sum of the events and complement should total 100%

$62 + 24 + 22 = 108\%$

NOT mutually exclusive

b)



b) $P(B \setminus L) = P(B) - P(B \cap L)$

$= 62 - 8$

$= 54\%$

c) $P(B \cup L) = P(B \setminus L) + P(L \setminus B) + P(B \cap L)$

$= 54 + 16 + 8$

$= 78\%$

If the probability of one event depends on the probability of another event, then these events are called dependent events.

If **event B depends on event A occurring**, then the conditional probability that event B will occur, given that event A has occurred, can be represented as follows:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

If event B depends on event A occurring, then the probability that both events will occur can be represented as follows:

$$P(A \cap B) = P(A) \cdot P(B|A)$$

Things to know:

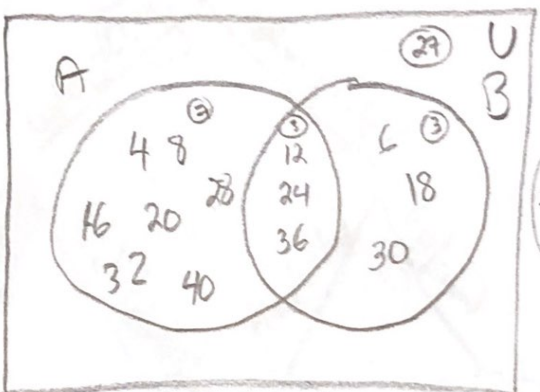
- A tree diagram is often useful for modelling problems that involve dependent events
- Drawing an item and then drawing another item, without replacing the first item, results in a pair of dependent events

Examples:

1. Nathan asks Riel to choose a number between 1 and 40 and then say one fact about the number. Riel says that the number he chose is a multiple of 4. Determine the probability that the number is also a multiple of 6, using each method below.

   a. A Venn diagram

   b. A formula



$= \frac{3}{10}$

A = {multiples of 4, 1 to 40}
B = {multiples of 6, 1 to 40}

b) $P(A \cap B) = P(A) \cdot P(B|A)$

$\frac{3}{40} = \frac{10}{40} \cdot P(B|A)$

$\frac{3}{40} \div \frac{10}{40} = P(B|A)$

$\frac{3}{40} \times \frac{40}{10} = P(B|A)$

$= \frac{3}{10}$

2. According to a survey, 91% of Canadians own a cellphone. Of these people, 42% have a smartphone. Determine, to the nearest percent, the probability that any Canadian you met during the month in which the survey was conducted would have a smartphone.

C = Cellphone
S = Smartphone

$P(C) = 0.91$

$P(S|C) = 0.42$

$P(S|C) = \frac{P(S \cap C)}{P(C)}$

$0.42 = \frac{P(S \cap C)}{0.91}$

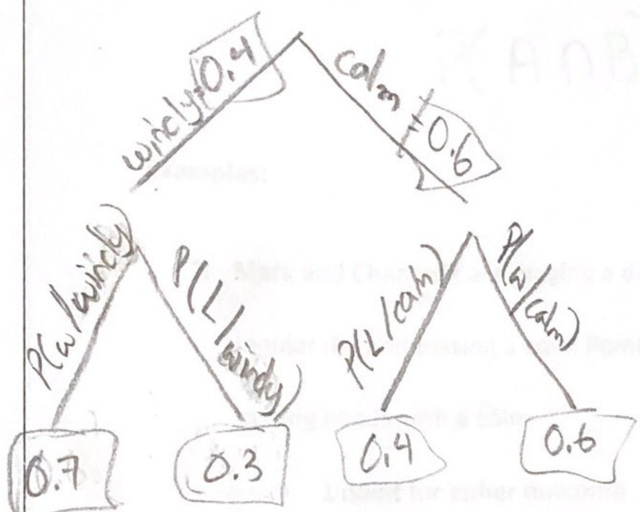
$\times$

$P(S \cap C) = 0.3822$

$= 38.22\%$

3. Hilary is the coach of a junior ultimate team. Based on the team's record, it has a 60% chance of winning on calm days and a 70% chance of winning of windy days. Tomorrow, there is a 40% chance of high winds. There are no ties in ultimate. What is the probability that Hilary's team will win tomorrow?

$P(\text{windy}) = 40\%$ $\quad P(\text{calm}) = 100\% - 40\% = 60\%$



$P(\text{win} \mid \text{windy}) = 70\%$

$P(\text{lose} \mid \text{windy}) = 30\%$

$P(\text{win} \mid \text{calm}) = 60\%$

$P(\text{lose} \mid \text{calm}) = 40\%$

$$P(\text{win}) = P(\text{windy} \cap \text{win}) + P(\text{calm} \cap \text{win})$$
$$= (0.4)(0.7) + (0.6)(0.6)$$
$$= 0.28 + 0.36$$
$$= 0.64$$
$$= 64\%$$

If the probability of event B does not depend on the probability of event A occurring, then these events are called **independent events.**

The probability that two independent events, A and B, will both occur is the product of their individual probabilities:

$$P(A \cap B) = P(A) \cdot P(B)$$

Examples:

1. Mark and Chantelle are playing a die and coin game. Each turn consists of rolling a regular die and tossing a coin. Points are awarded for a rolling 6 on the die and/or tossing heads with a coin:

   - 1 point for either outcome
   - 3 points for both outcomes
   - 0 points for neither outcomes

   Players alternate turns. The first player who gets 10 points wins.

   a) Determine the probability that Mark will get 1, 3, or 0 points on his first turn.

S = rolling a six

H = tossing a heads

$P(S) = \frac{1}{6}$ $P(H) = \frac{1}{2}$

$P(S') = \frac{5}{6}$ $P(H') = \frac{1}{2}$

$P(S \cap H) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

$P(S \cap H') = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

$P(S' \cap H) = \frac{5}{6} \times \frac{1}{2} = \frac{5}{12}$

$P(S' \cap H') = \frac{5}{6} \times \frac{1}{2} = \frac{5}{12}$

$P(\text{score } 3) = \frac{1}{12}$

$p(\text{score } 0) = \frac{5}{12}$

$p(\text{score } 1) = \frac{5}{12} + \frac{1}{12} = \frac{6}{12}$ or $\frac{1}{2}$

b) Verify your results for part a). Explain what you did.

$$\frac{1}{12} + \frac{5}{12} + \frac{6}{12} = \frac{12}{12} \checkmark$$

2. All 1000 tickets for a charity raffle have been sold and placed in a drum. There will be two draws. The first draw will be for the grand prize, and the second draw will be for the consolation prize. After each draw, the winning ticket will be returned to the drum so that it might be drawn again. Max has bought 5 tickets. Determine the probability, to a tenth of a percent, that he will win at least one prize.

X = winning grand prize

Y = winning consolation prize

Z = winning at least one prize

$P(X) = \frac{5}{1000} = 0.005$

$P(Y) = \frac{5}{1000} = 0.005$

$P(X') = 0.995 \quad P(Y') = 0.995$

$P(X \cap Y) = 0.005 \times 0.005$
$= 0.000025$

$P(X \cap Y') = 0.005 \times 0.995$
$= 0.004975$

$P(X' \cap Y) = 0.004975$

$P(X' \cap Y') = 0.990025$

$P(\text{win } 2) = 0.000025$
$+$
$P(\text{win } 1) = 0.004975 \times 2 = 0.0095$

$P(Z) = 0.009975$
$\approx 1\%$